The use of the Manning equation is not safe for different river styles. What are the alternatives?

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Abstract

The element of flow resistance is given distinctive attention due to its significant role in the assessment of the sediment entrainment processes, flood risk and management, and the effective design of water resources-related projects. Several flow resistance equations are well-acknowledged and they do not need to be calibrated when the uniform flow is deemed for a river reach. The Manning n formula is the best-known and most widely-used equation. Nevertheless, a question is raised here "Is it safe to use the Manning equation for different river styles?". This commentary article answers this question by taking different sources of resistance to flow (including bed-sediment size, bed forms, vegetation, etc.) for different river styles seen in lowland (mostly sand-bed rivers) and mountainous areas (mostly gravel-bed rivers). The article interprets the previous literature and large field data sets in different river styles and reveals that the Manning equation is rarely a good choice for the calculation of flow resistance due to its variations with river flow discharge. This will bring adverse implications into many hydrological and geomorphological applications e.g. in geomorphological modeling since small differences in the prediction of flow depth result in big differences in the simulated bed-load transport rate. Collectively, the use of the Manning equation is limited to deep flows with uniform plain beds and less form resistance. The alternatives for the Manning equation are logarithmic and power equations using relative submergence and flow discharge approach which are illustrated in detail and evidence is presented to show their better accuracy and wider applicability. Finally, the flowchart for identifying alternative resistance laws (IARL) is presented to facilitate the selection of resistance laws for different river styles.

Key words: Alternative resistance laws, Flow discharge, Flow resistance, Manning equation

Abstrakt:

Zložke hydraulického odporu sa venuje osobitná pozornosť vzhľadom na jeho významnú úlohu pri hodnotení procesov zanášania sedimentmi, povodňového rizika a manažmentu povodní a efektívnom navrhovaní projektov súvisiacich s vodnými zdrojmi. Poznáme niekoľko rovníc hydraulického odporu a nie je potrebné ich kalibrovať, keď sa predpokladá rovnomerný prietok vo vodnom toku. Manningov vzorec n je najznámejšou a najpoužívanejšou rovnicou. Napriek tomu existuje otázka: "Je bezpečné použiť Manningovu rovnicu pre rozličné vodné toky?". Tento článok odpovedá na túto otázku tým, že berie do úvahy rôzne zdroje hydraulického odporu voči prúdeniu (vrátane veľkosti koryta, formy koryta, vegetácie atď.) pre rôzne vodné toky pozorované v nížinách (väčšinou vodné toky s pieskovým korytom) a horských oblastiach (väčšinou vodné toky so štrkovým korytm). Článok interpretuje dostupnú literatúru a veľké súbory terénnych údajov v rozličných korytách vodných tokov a odhaľuje, že Manningova rovnica je zriedka dobrou voľbou na výpočet hydraulického odporu kvôli jej zmenám s prietokom vodného toku. Tento fakt môže priniesť nepriaznivé dôsledky do mnohých hydrologických a geomorfologických aplikácií, napr. v geomorfologickom modelovaní, pretože malé rozdiely v predikcii hĺbky vodného toku vedú k veľkým rozdielom v simulovanej rýchlosti transportu sedimentov. Súhrnne je použitie Manningovej rovnice obmedzené na hlboké vodné toky s rovnomernými korytami a menšími prekážkami v dne toku. Alternatívy pre Manningovu rovnicu sú logaritmické a empirické rovnice využívajúce prístup relatívneho prietoku, ktoré sú podrobne ilustrované a sú prezentované dôkazy, ktoré ukazujú ich lepšiu presnosť a širšiu použiteľnosť. Nakoniec je uvedený vývojový diagram na identifikáciu alternatívnych vzťahov hydraulického odporu, aby sa uľahčil výber vzťahu hydraulického odporu pre rôzne druhy vodných tokov.

Kľúčové slová: Alternatívne vzťahy odporu, Prietok, Hydraulický odpor, Manningova rovnica

1 Commentary

There is always resistance to flow by frictional forces retarding the flow of a river. For any given channel geometry, channel slope, and flow discharge an increase in frictional resistance causes deeper and slower flow, with consequences for flood risk, aquatic habitat, shear stress and bed-material transport (Ferguson, 2007). The sources of flow resistance in rivers can be from any feature of the channel topography, shape or bed texture (grain size and bed forms) that serve to retard the river flow and dissipate flow energy. Generally, any feature in the river channel (friction with the bed particles or banks, bed forms, vegetation, etc.) acts as a form of flow energy dissipator and should be taken into account for the calculation of the flow resistance. For many geomorphological purposes and modeling, the main flow properties of a river scale including water depth (or hydraulic radius, R), velocity (u), wetted cross-sectional area (A) and shear stress are required to quantify flow resistance. There are many equations, but by far the most well-known and widely used equation is what is known as the Manning equation calibrated for uniform flow (Okhravi et al. 2022).

$$u = \frac{1}{n} R^{(2/3)} S^{(1/2)} \tag{1}$$

where n is the roughness coefficient (Manning number) with a dimension of $L^{-1/3}T$, R is the hydraulic radius (L), and S is the bed slope or energy slope (dimensionless).

According to the originated conditions of the Manning equation, n is constant for a river reach. This means that n is invariant with the river flow stage and it can be calibrated from flow field measurements at one time, then used to predict past or future conditions (served as the classic use of the Manning equation e.g. for flood discharge). The more recent application of the Manning equation was in the prediction of water surface level in one-dimensional (1D, width-averaged) and two-dimensional (2D, depth-averaged) numerical hydrodynamic and morphodynamic models. The Manning equation is usually the default equation, often the only equation, in many hydrodynamic models and numerical codes. For instance, the well-known HEC-RAS model uses the fixed value of Manning resistance calibrated to water levels at just one discharge for hydrodynamic simulations at all places and times. The users need to assign n based on the engineering experience (expert judgment) or estimate this from the channel properties. The sediment size of the bed is the first obvious characteristic to consider, while the flow resistance depends on more than sediment size (e.g. bed forms). The most widespread method is to use Strickler's relation for estimating n (Eq. 2). In this equation, d_{50} is the median grain size of the bed and d_{84} stands for the sediment size that is 84% finer.

$$n \approx 0.047 d_{50}^{1/6} \approx 0.039 d_{84}^{1/6} \tag{2}$$

The goal of this commentary is to illuminate the title of the article and answer the concerning question. This article persuades hydraulic/hydrologic researchers and engineers and geomorphologists that the Manning equation is rarely a good choice for the calculation of flow discharge and velocity, despite its popularity. The previous literature showed that n is not constant with the change of flow discharge and it tends to decrease significantly as flow discharge increases (Chow, 1959; Dingman, 2009; Okhravi et al. 2022; Ferguson et al. 2022). Ferguson (2010) used large data sets from more than 20 publications to investigate the variances of flow resistance with changes in flow discharge, excluding the overbank flows and reaches highly covered with vegetation and wood debris accumulation. The research illustrated that n values are highly variant with stage discharge in both gravel-bed and sand-bed rivers. Collectively, the Manning equation or the combination of the Manning and Strickler equations (hereafter referred to Manning-Strickler) usually underestimates flow resistance even in high flows, and it brings adverse implications in many hydrological and morphological applications.

To understand the application of the Manning equation in different river styles e.g. gravel-bed and sand-bed rivers, the basic differences between the mentioned rivers are briefly explained. Rivers are mainly categorized by the bed gradient and the sediment size range. Gravel-bed rivers are mostly located in mountainous areas having high gradients (> 0.002 m/m) and are dominated by gravel, cobble or boulder bed particles. On the other side, the morphological appearance of sand-bed rivers or low-gradient rivers shows a river bed dominated by sand-size particles (Okhravi, 2022). The review in the literature revealed that the Manning equation had flaws in the prediction of flow resistance on relative submergence $R/d_{84} < 10\text{-}20$ (Ferguson, 2010) for both mentioned river types. The discrepancies between measured and computed flow resistance values originate from the basic conditions used to fit the Manning-Strickler equation for relatively deep flows over uniform beds with less form resistance than many natural rivers. Whilst, gravel-bed rivers usually experience low relative submergence ($R/d_{84} < 4$) and non-uniform sediment mixtures. On the other hand, sand-bed rivers ($R/d_{84} > 10$) are characterized by lower regime flow bed forms, dune (over coarse sand), and ripple (over fine-medium sand), which provide significant form resistance. Additionally, the use of fixed n value for flow resistance in sand-bed rivers is deemed practically questionable in the presence of a mobile bed and time-dependent changes in vegetation patches (Okhravi et al. 2022).

Several alternatives are available rather than Manning and Manning-Strickler equations which better represent flow resistance variations within a reach. The comprehensive study of Fergusen (2007) on the performance of several flow resistance equations for the prediction of water velocity or discharge in different rivers showed that the Manning-Strickler predictions fit the measured values by a factor of two (i.e. the predictions were within ½ and twice the measured flow resistance) in only almost 50% of cases while the logarithmic and power law approaches (Eqs. 3 and 4, respectively) using relative submergence scaled on a d_{84} showed better accuracy in more than 75% of cases. The formulations of the second well-known flow resistance relations connecting dimensionless Darcy-Weisbach friction factor ($f = 8(u_*/u)^2$, reference equation for calculating Darcy-Weisbach friction) to relative submergence are presented below (Afzalimehr et al. 2011; Okhravi and Gohari, 2020):

$$\frac{u}{u_*} = \sqrt{\frac{8}{f}} = a_1 \log\left(\frac{R}{d_{84}}\right) + a_2 \tag{3}$$

$$\frac{u}{u_*} = \sqrt{\frac{8}{f}} = \alpha\left(\frac{R}{d_{84}}\right)^{\beta} \tag{4}$$

where a_1 and a_2 (as well as α and β) are empirical constant coefficients. u_* denotes shear velocity dimensioned by LT^{-1} and calculated with $(gRS)^{1/2}$. g is the gravitational acceleration.

The discussion has so far shown that Manning and Manning-Strickler equations with n estimated by only the particle size of the bed or calibrated at a single flow cannot represent how flow velocity and flow discharge vary with the flow stage, leading to inaccurate predictions of flow depth variations. This has significant effects on morphological modeling, that small differences in water depth can lead to big differences in the predicted bed-load transport rate. Continually, this will provide errors in the morphodynamic modeling that they use the bed-load transport to compute the bed elevation updates during simulation time. This all turned to the importance of a well-accurate prediction of flow resistance.

As briefly explained before, the effects of vegetation and bed forms and other resistance sources were neglected, while low-gradient sand-bed rivers were usually occupied with aquatic vegetation and bed forms (dune and ripple). Since the Manning and Manning-Strickler equations do not take vegetation and bed forms impacts into account, the logarithmic flow resistance could be the alternative. According to the review study by Sulaiman et al. (2017) in lowland rivers, they showed that the relative submergence is usually high, and sediment motion is only initiated with low shear stress. Hence, the effects of relative submergence on the sediment transport rate can be neglected. Moreover, the well-distributed fine uniform sediments consist of the bed in lowland rivers such that the median particle size (d_{50}) is usually transported as the suspended loads. Therefore, for sand-bed rivers, the main sources of boundary roughness are the lower regime bed forms and the vegetation occurrence. Hence, the resistance law using relative submergence is not often the safest choice since it does not account for vegetation. The best alternative to predict flow resistance in sand-bed rivers dominated by vegetation is nondimensional hydraulic geometry equations that connect the dimensionless mean flow velocity (u^{**}) and dimensionless unit discharge (q^*) (Rickenmann and Recking, 2011). A recent study by Okhravi et al. (2022) showed that the flow discharge is the only characteristic that shapes the channel and postulates fully the bed forms (including irregular bed topography and water elevation variations) and submerged vegetation occurrence. Therefore, the relationships based on flow discharge have higher reliability than those which are according to relative submergence (Eqs. 3, 4) in sand-bed rivers covered by vegetation. The developed form and optimal parameterization of such a relationship from a wide range of field data sets of four lowland streams near Bratislava in Slovakia have been proposed by Okhravi et al. (2022):

$$u^{**}= 0.978q^{*0.953} \tag{5}$$

In Eq. (5), constants are determined empirically by regression analysis following the defined $u^{**}= u/u_* = u/(gRS)^{0.5}$ and $q^*= q/(gR^3S)^{0.5}$; q is the flow discharge per unit channel width ($q = Q/B$) and B is the channel width.

A comparison of the predicted and measured flow resistance values indicated that 87-89% of the data sets were within ±20% error bands (Fig. 1a). According to the results of error analysis, the new predictor (Eq. 5) indicated a good performance because of the high value of I_a and the low values of RMSE (root mean square error) and SI (scatter index). The flow resistance predictor was also verified against large sets of collected field data at a lowland river located at the Upper Stör catchment in Northern Germany. The obtained predictions using the developed predictor might overestimate flow resistance factors by 40% for other lowland rivers.

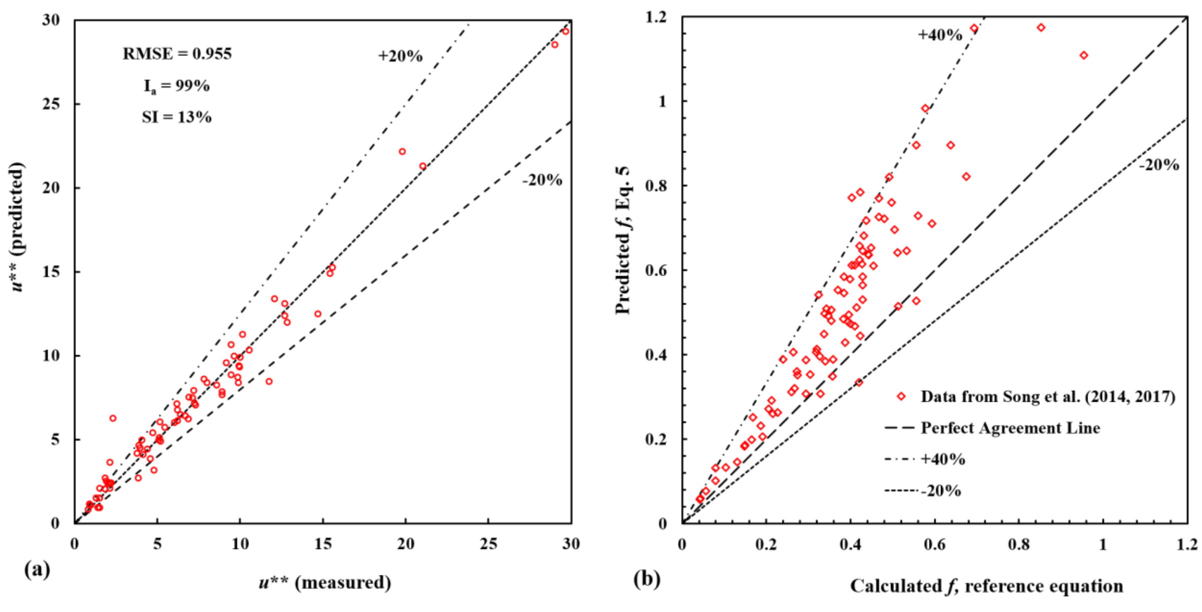

Figure 1: The performance of the new flow resistance predictor through measured field data (a) and its verification through collected field data from a lowland river with the same physiographic region at the Upper Stör catchment in Northern Germany (b)

This commentary manifests that the Manning-Strickler equations are not safe for different types of rivers in all spaces and times and geomorphologists must be cautious about applications of these equations. The presented evidence suggests these equations do not provide accurate predictions generally in steep and small rivers, shallow flows and the presence of vegetation and bed forms. However, the detailed conditions have not previously been elucidated and the author is trying to make it more discernible through the proposed flowchart for identifying alternative resistance laws (IARL)

(Fig. 2). The IARL flowchart reveals that the Manning-Strickler equations can be used in limited conditions in deep rivers and they are just reliable for fine plane gravel beds or medium/fine ripples in sand beds, but not for dune beds and big ripples or flows over vegetation. In other cases, logarithmic and power equations and flow discharge approaches are preferable. The logarithmic or power equation is often the safest choice for the prediction of flow resistance since they usually cover the required range of relative submergence even in relatively shallow flows. In conclusion, the Manning and Strickler equations should bed used with great caution and should not be default equations in geomorphological practice and modeling.

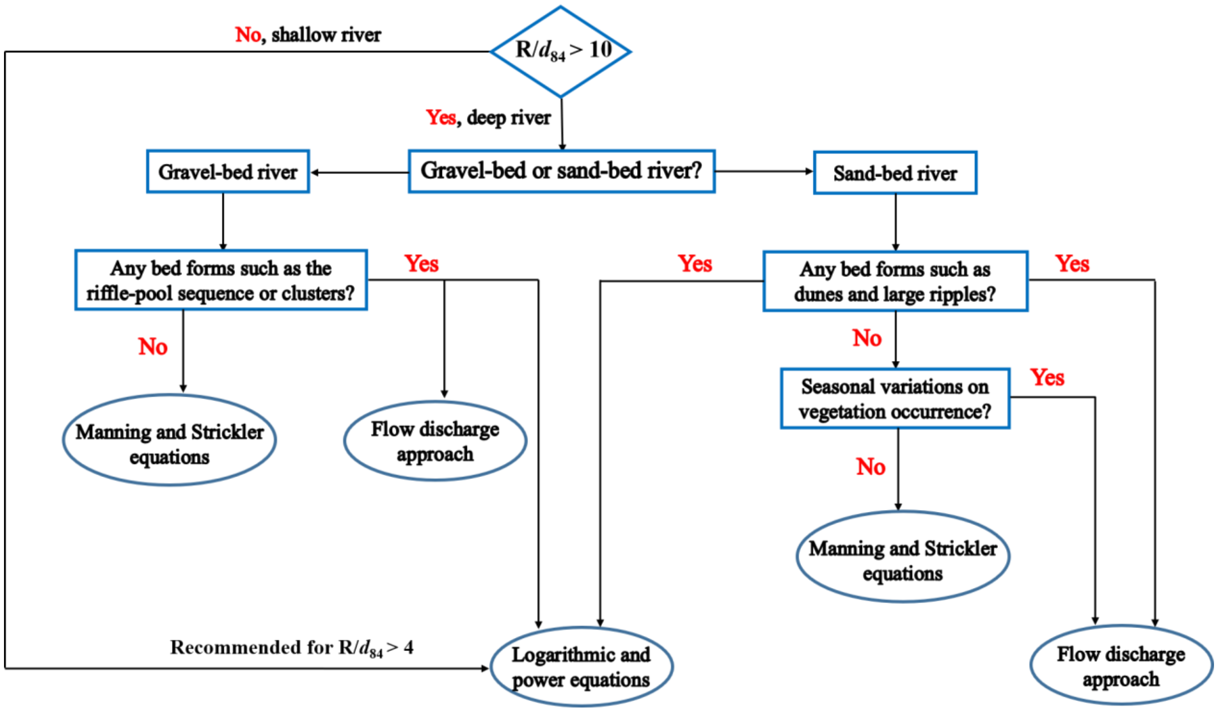

Figure 2: Flowchart of the proposed identifying alternative resistance laws (IARL)

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