MODELING OF UNSTEADY SEDIMENT TRANSPORT AT THE HIGH SHEAR STRESS IN FLUME.

Štěpán Zrostlík[1,a] and **Jan Krupička**[1]

[1]Department of Hydraulics and Hydrology, Czech Technical University in Prague, 166 29 Prague 6
[a]Corresponding author: stepan.zrostlik@fsv.cvut.cz

Annotation

This paper is concerned with the construction of simple mathematical model of unsteady sediment motion in flume. The model is based on quasi-steady solution of flow and balancing of erosion and deposition at the each time step. We used two variation of transport equation and two variation of friction equation. One friction equation was derived for this application. Model date was compared to measured date from laboratory flume. The comparison experiments were under steady and unsteady sediment transport condition. The deviation are acceptable for measured and compute date.

Key words

Mathematical model, intense sediment transport, flume experiment

INTRODUCTION

Human activities and natural processes can result in an unbalance between water and sediment discharges in rivers. Many experts deal with the phenomenon of sediment transport (e.g. Wiberg, 1989). In the most of experimental studies devoted to the sediment transport, rather low intensity of sediment motion was a characteristic condition. The matter is relatively well understood under these conditions. On the other hand, significantly less experimental effort was devoted to investigation of flows in which the sediment discharge takes more than, say, 5% of the total discharge. Inner structure and flow patterns of such flows are the subjects of continual research (e.g. Matoušek, 2010). Intense transport of coarse sediment develops in flow with free water surface if the energy grade line is steep and the flow produces high bed shear stress. This mode is typical for floods and mountain streams. For better prediction and protection of people and structures it is necessary to develop appropriate mathematical models.

In principle, two different approaches are employed in modeling of sediment transport in steep streams. The first one uses hydro-sedimentologic models considering sediment transfer processes on catchment basin. DHVSN model (Doten et al., 2006) is an example. The second approach uses 1D or 2D hydrodynamics simulations including sediment transport and accounting for variations in bed geometry due to erosion or deposition. There are many models designed for simulation of flow of water and sediment (eg. Termini, 2013) but less for really intense sediment transport. An example is model SETRAC (Chiari et al., 2010) designed for mountains streams.

Up to now, many empirical formulae were proposed for prediction of transport capacity and bed friction of sediment laden flow. Performance of some of them was discussed by Šulc and Zrostlík (2013) on this conference. The objective of this study is to test selected formulae against new experimental data collected in a laboratory flume and to implement these equations into a simple model of unsteady sediment motion. As the computational principles are the same in the flume and in rivers, successful validation of the model on flume data would open the door to extension of the model to field of river hydraulic.

FRICTION AND TRANSPORT FORMULA

Several friction and transport formulae were compared with preliminary experimental data by Šulc and Zrostlík (2013). The best predictions were produced by transport formula (1a) by Rickenman

(2001), where the critical value of transport rate was calculated using correlation (1b) by Whittacker and Jäggi (1986). Equation (2) by Rickenmann and Recking (2011) seemed to be the best working friction formula. These formulae were selected to be tested on our new experimental data.

$$q_s = 3.1 \cdot \left(\frac{d_{90}}{d_{50}}\right)^{0.2} \cdot (q_m - q_c) \cdot I_e^{\,1.5} \cdot (s-1)^{-1.5} \qquad (1a)$$

$$q_c = 0.143 \cdot g^{0.5} \cdot d_{90}^{\frac{1}{5}} \cdot I_e^{\,-1.167} \cdot (s-1)^{-1.5} \qquad (1b)$$

$$V_m = (I_e \cdot h \cdot g)^2 \frac{6.5 \cdot 2.5 \cdot \frac{h}{d_{84}}}{\sqrt{6.5^2 + 2.5^2 \cdot \left(\frac{h}{d_{84}}\right)^{\frac{5}{3}}}} \qquad (2)$$

Friction formula of the Meyr-Peter and Muller type (eq. 3) is very frequent in literature and many researchers proposed specific values of formula coefficients to fit their experimental data. Matoušek (2009) extended validity of this type of formula to high shear stresses and he proposed more general form by relating the coefficients α and β to particle properties characterised by particle Reynolds number Re_P (eq. 4).

$$\Phi_E = \frac{q_s}{\sqrt{(\rho_s / \rho_f - 1) \cdot g \cdot d_{50}^{\ 3}}} = \alpha \cdot \left(\theta_b - \theta_{cr}\right)^{\beta} \qquad (3)$$

$$\alpha = 5.4 + \frac{58}{Re_P^{\ 0.62}}, \qquad \beta = 1.2 + \frac{1.3}{Re_P^{\ 0.39}} \qquad (4)$$

Although calibrated on data from pressurized pipes, better performance of eqs. (3, 4) is clearly demonstrated by our flume data plotted in figure 1a. Our data were collected in a new tilting flume in the Laboratory of Water Engineering of Czech Technical University in Prague. The flume is 0.2 m in width and the mixture of water and sediment is recirculated by pipe with centrifugal pump. Boundary conditions were set to ensure uniform steady flow with intense sediment transported above flat granular bed without bed forms. Particles were glass spheres of 3 and 1.5 mm in diameter. The measured quantities were discharge of mixture, delivered concentration, flow depth and slope of water and bed surface.

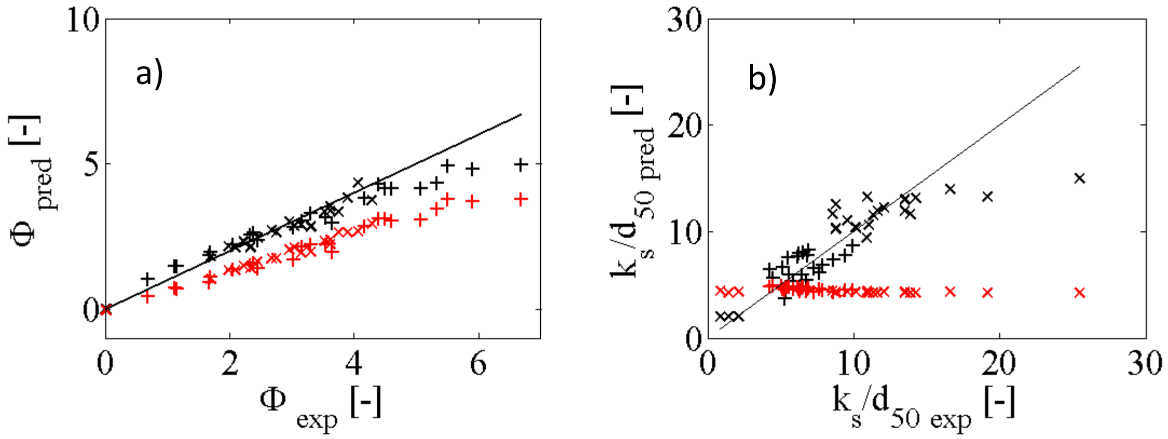

Figure 1: *++ d_{50} = 1.5mm, × d_{50} = 3mm.*
a) Transport of sediment: +× eqs. (3, 4), +× eq. (1).
b) Equivalent roughness of bed: +× eq. (5), +× eq. (2).

Surface of eroded bed with intense sediment transport above the bed is often treated as a hydraulically rough boundary (e.g. Yalin, 1992). Then the bed can be characterised by an equivalent roughness and a suitable friction law (e.g. that by Nikuradse) can be employed to calculate friction factor. Reversely, hydraulic roughness can be calculated form experimentally determined hydraulic gradient. At abscissa in figure 2b, hydraulic roughness evaluated from our experiments and normalised by particle size is plotted. Normalised hydraulic roughness predicted by eq. (2) in combination with Nikuradse's friction law is at abscissa. It is seen, that k_s / d_{50} is strongly underestimated. High shear stress experiments conveyed in pressurized ducts revealed that k_s / d_{50} is closely related to Shields parameter of bed (e.g. Wilson, 1984). Recent studies indicated that additional parameters should be taken to account. Introduction of dimensionless particle velocity $V_t{*} = V_t \cdot \sqrt[3]{\left(\rho_s / \rho_f - 1\right)^2 / \left(g \cdot \nu_f\right)}$ seemed to improve correlation significantly (Camenen, 2006). We calibrated eq. (5) to fit our data with a result plotted in the figure 1b.

$$k_s / d_{50} = 0.63 \cdot V_t{*}^{1.40} \cdot \theta_b^{0.74} \qquad (5)$$

PREDICTION OF HYDRAULIC GRADIENT AND DELIVERED CONCENTRATION

An important advantage of friction formula (2) is that hydraulic gradient is calculated directly from mean velocity and water depth. Formulae employing Shields number have to be solved in combination with friction law and Darcy-Weisbach equation to produce hydraulic gradient. Because the Shields parameter alone is related to hydraulic gradient, the solution is iterative (see flow chart in the figure 3a). Prediction of hydraulic gradient and delivered concentration (defined as ratio of the sediment discharge and the total discharge) is compared with measured data in figures 2. It is seen that procedure based on eqs. (3, 4, 5) produce somewhat better predictions, but the advantage over formulae proposed by Rickenmann is not as strong as in the case of hydraulic roughness. This observation indicates that inaccuracy in prediction of k_s / d_{50} do not affect evaluation of I_e significantly. Moreover, the above mentioned iterative solution often suffers by numerical problems – the phenomenon discussed recently by Camenen (2013). In the light of these findings, application of correlations for k_s / d_{50} becomes rather questionable, at least in case of open channel flow.

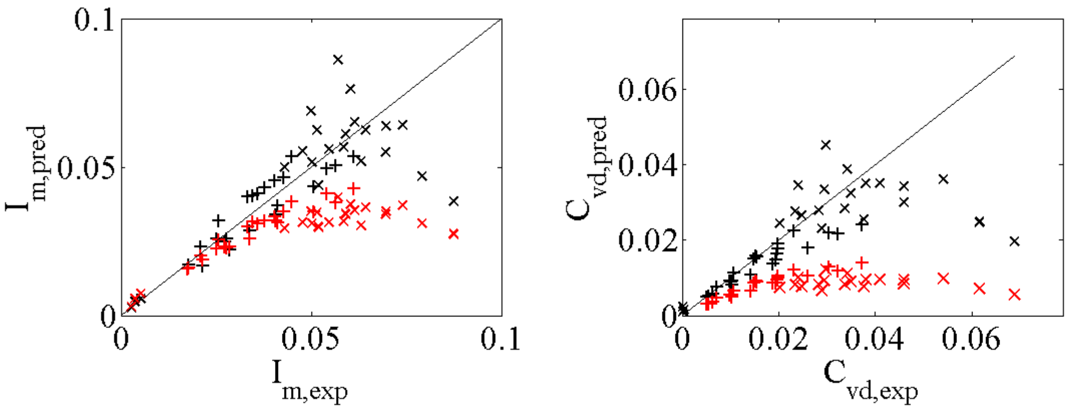

Figure 2: *Hydraulic gradient and delivered concentration. Comparison of experimentally determined values with numerical prediction. +× eq. (1) and (2), +× eq. (3,4,5) in combination with Nikuradse's friction law. ++ d= 1.5mm, ×× d= 3mm.*

MODEL OF UNSTEADY SEDIMENT MOTION

The model is designed for slowly changing flows. In other words, boundary conditions are assumed to vary with the time scale larger then is the time period in which the flow became steady after the boundary condition has been changed. This assumption allow us to consider the flow quasi-steady and to calculate water surface profile employing the standard step method at the each time. The space domain is discretized into sections by a sequence of cross-sections (figure 4). Initial condition is profile of bed positions. At the each time step, the boundary conditions are 1) discharge of water and discharge of sediment in the upstream cross-section (in direction of flow), 2) bed level in the downstream cross-section, and 3) depth in upstream/downstream cross-section for the supercritical/subcritical flow respectively. Iterations are required to found bed and water surface profile in the new time step. Initial estimation of sediment discharge is used by the first iteration. Then the change of bed level is calculated for the each cross-section by balancing erosion (or deposition) and the sediment discharges within the preceding time interval. For the new bed positions, water surface profile is calculated by the step method employing selected friction formula. Then a new estimation of sediment discharge is calculated for the each section from the mean depth and slope of energy grade line. The new estimation of sediment discharge is used in the next iteration until a convergence is achieved. Flow chart of iterative procedure is shown in the figure 3b.

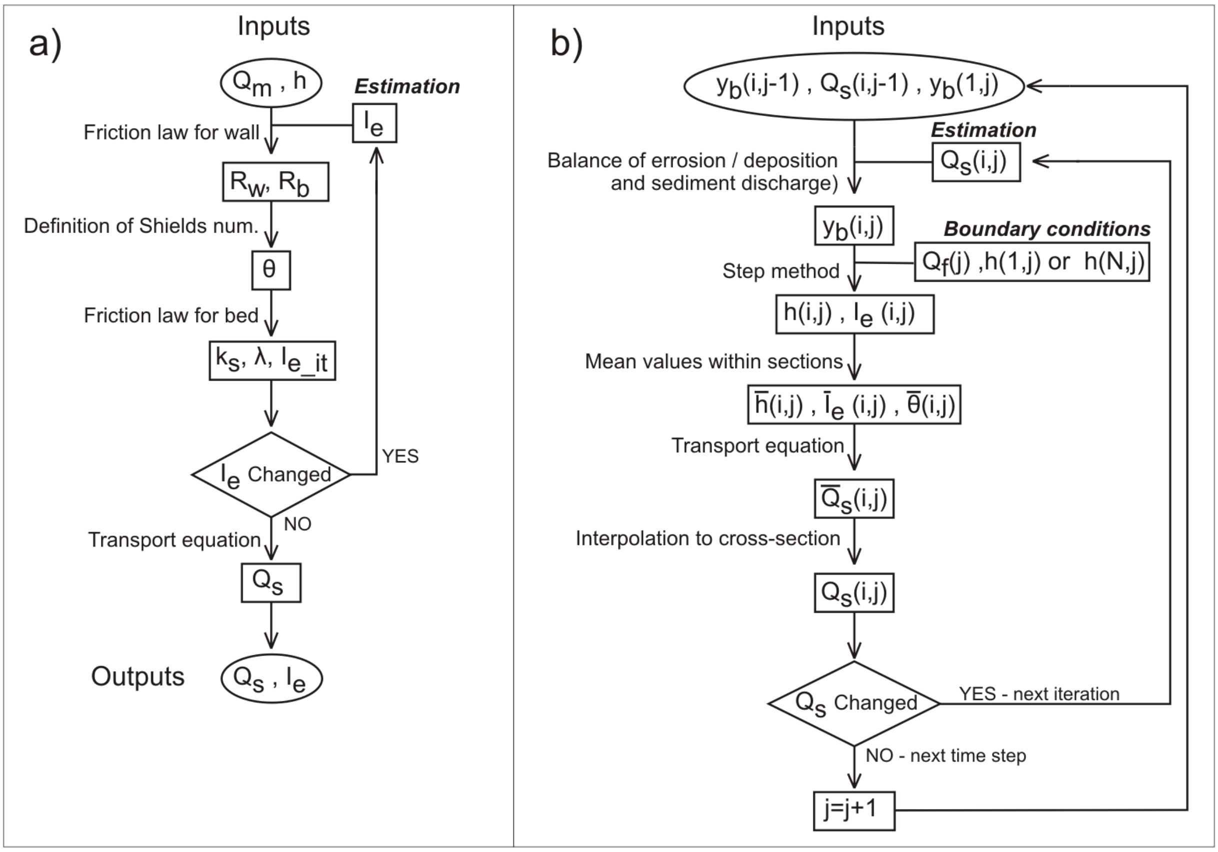

Figure 3: *a) Flow chart of iterative evaluation of hydraulic gradient followed by calculation of discharge of sediment. b) Flow chart of calculation of bed and water surface profile in one time step of model of unsteady sediment motion.*

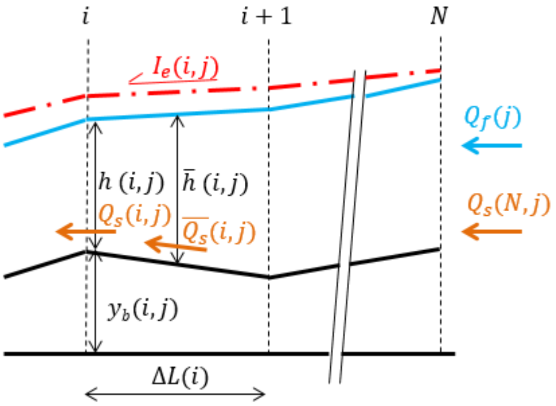

Figure 4: *Scheme of numerical discretization used in model of unsteady sediment motion.*

To test the model performance, an experiment on unsteady sediment motion was carried out in our flume. The flow was uniform and steady at the beginning of experiment. The unsteady sediment motion was induced by manipulation with overshot weir down at the low end of the flume. This caused increase of sediment transport and erosion of deposit at the low section of flume. Because of recirculating system, the erosion at the low end resulted in increasing sediment discharge at the inlet to the flume. Total discharge and delivered concentration were measured in pipe of recirculating system. Position of bed was observed visually and position of water surface was sensed by ultrasound gauge at upper end of flume, at lower end of flume and at two interlaying cross-sections. Length of record was 5 minutes. The steady state at the beginning of experiment served as initial conditions for the numerical simulation. Boundary conditions for the model were measured position of bed at the lower end of flume, measured depth at the upper end of flume (the flow was supercritical), and discharges of water and sediment at inlet.

Figures 5 show results of two numerical simulations in a form of longitudinal profiles of bed and water surface. Friction and transport formulae (1) and (2) were used in the first simulation (dashed line) whereas formulae (3), (4) and (5) were used to produce the solid lines. Three time steps are plotted. Considering the first time step at $t = 0$ s (with the bed surface given by initial condition), both sets of formulae provides good prediction of water surface profile, but formulae proposed by Matoušek in combination with our new correlation works slightly better. Position of bed is overestimated by both simulations in next two snapshots (taken in the middle and at the end of experiment). Observed discrepancy can be partially explained by improper boundary condition at inlet – sediment flow rate is measured in recirculating pipe and it is assumed that the same flow rate enters the flume at inlet, but there can be some deviation because of sediment accumulation in connecting piping. However, simulation employing eq. (3), (4) and (5) provides somewhat better prediction again.

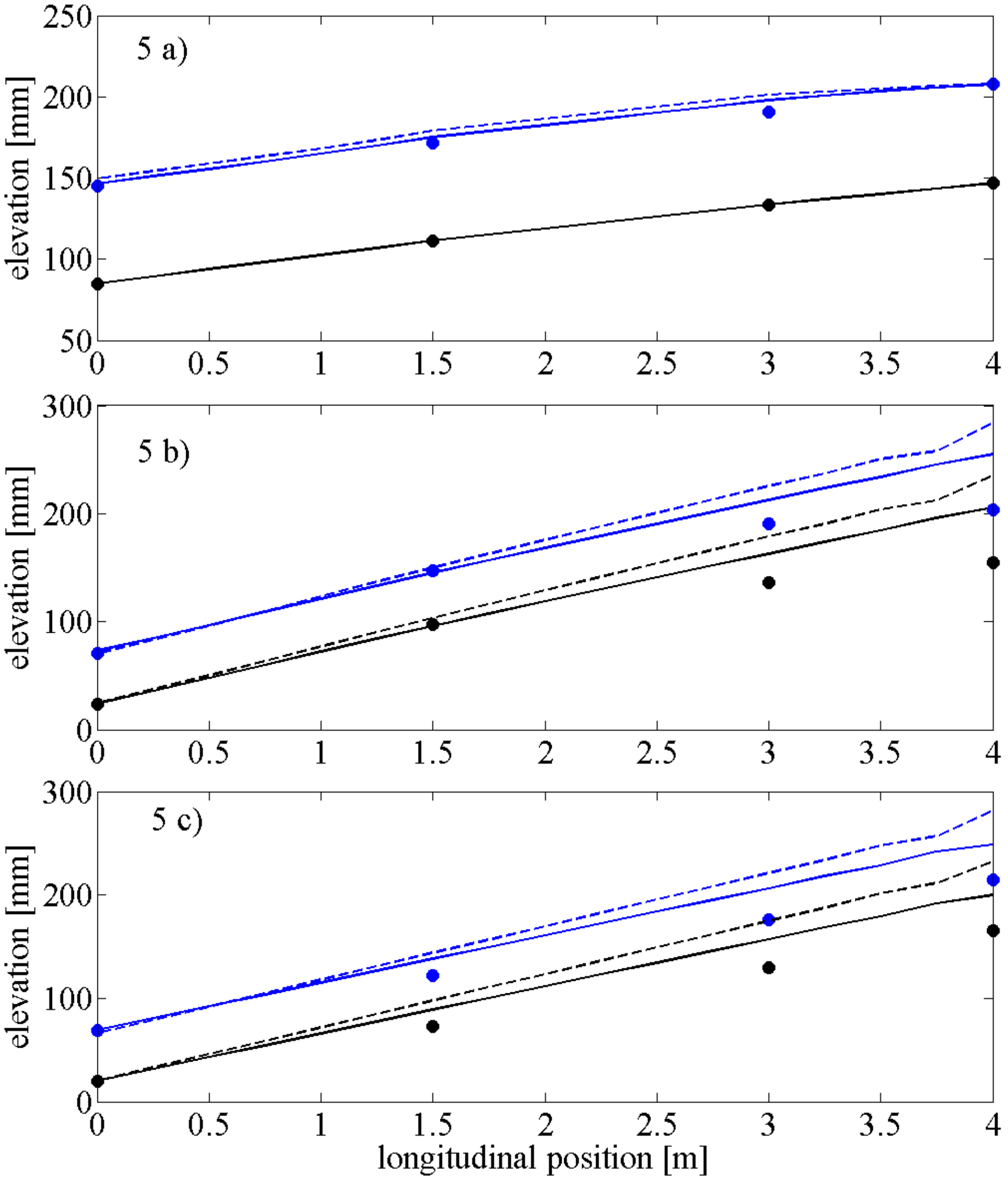

Figure 5: *Scheme of numerical discretization used in model of unsteady sediment motion.*

CONCLUSION

Performance of friction formula proposed by Rickenmann (2011) was tested using new data on flume flow with intense transport of sediment. An alternative formula based on correlation with Shields parameter was calibrated on our data. Although Rickenmann's formula failed when used to predict hydraulic roughness of eroded bed, prediction of hydraulic gradient was not considerably worse than prediction based on the alternative formula. Moreover, an advantage of friction formulae based on correlation with Shields parameter is doubt by necessity of iterative solution which often suffers by numerical problems.

Predictive formulae for transport of sediment were also tested. Considering our data, formula proposed by Matoušek (2009) worked considerably better than formulae proposed by Rickenman (2001), Whittacker and Jäggi (1986).

All above formulae were implemented in model of unsteady motion of sediment. The model is based on quasi-steady solution of water flow in each time step, and on balancing erosion and sediment discharges within interval between time steps. Preliminary test of the model performance was carried out using experimental data on unsteady sediment motion. Profile of water depth was successfully predicted but calculated profile of bed position diverged markedly from the measured one at the end of simulation. Refinement of the model is subject of on-going work.

LIST OF SYMBOLS

Indices s, f, m – solid, fluid, mixture
d – diameter of particle
g – gravitation acceleration
h – depth of flow
i – index of cross-section
I_e – energetic gradient
j – index of time step
k_s – hydraulic roughness
y – position of bed
q – specific discharge
Q – discharge
Re_p – particle Reinolds number
R_w – hydraulic radius associated with wall
R_b – hydraulic radius associated with bed
V_t – terminal sedimentation velocity
λ – friction factor
ρ – density
ν – kinematic viscosity
Θ – Shields parameter
Θ_{cr} – critical Shields parameter
Φ – Eistein number

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Štěpán Zrostlík, Jan Krupička: MODELOVÁNÍ NEUSTÁLENÉHO CHODU SEDIMENTU VE ŽLABU ZA VYSOKÝCH SMYKOVÝCH NAPĚTÍ.

Abstrakt

V dnešní době, kdy se začínají vyskytovat častěji bleskové povodně a vlivem výstavby se zrychlují odtoky z urbanizovaných ploch, je problematika chodu sedimentu aktuální. Pro lepší předpověď a ochranu ohrožených území je nutno vytvoření modelu popisující tento fenomén. V této práci se zabýváme vytvořením matematického modelu popisujícího intenzivní chod sedimentu v laboratorním žlabu. Princip transportu chodu sedimentu je stejný ve žlabu jako v tocích a může být poté implementován do říčních modelů.

Model předpokládá pomalé změny v poloze dna vůči změně průtoku v časovém kroku, zároveň počítá bilanci sedimentu ve výpočetním úseku. Vytvořený matematický model umožňuje vlastní volbu transportní a drsnostní rovnice. V současné době je odvozeno mnoho těchto rovnic. Námi byly vybrány a testovány dvě transportní rovnice (Matoušek 2007 a Rickenmann 2011) a dvě drsnostní rovnice. Rovnici již dříve testovanou (Rickenmann Recking 2001) a námi kalibrovanou rovnici složitějšího tvaru s explicitním vyjádřením drsnosti. Model je jedinečný v přístupu řešení rovnic. Oproti ostatním řeší současně množství transportovaných částic a drsnost způsobenou samotným transportem. Výpočet v každém prostorovém i časovém kroku vyžaduje několik iterativní úrovní pro vyřešení soustavy rovnic.

Žlab, na kterém byly prováděny experimenty pro porovnání, je postaven jako sklopný s erodovatelným dnem. Dopravu sedimentu do žlabu zajišťují odstředivá čerpadla. Součástí každého experimentu bylo měření průtoku směsi, dopravní koncentrace částic, měření podélného profilu hladiny a sedliny ve žlabu. Při prováděných experimentech byly použity skleněné kuličky průměru 1,5 a 3 mm.

Správnost vybraných transportních a drsnostních rovnic byla ověřována na změřených ustálených stavech. Bylo zjištěno, že nejvíce záleží na volbě drsnostní rovnice. Lepších výsledků bylo dosahováno se složitějším tvarem drsnostní rovnice, která obsahuje závislost na Shieldsově čísle. U druhé rovnice docházelo k podhodnocování předpovídaných hydraulických sklonu. Což vedlo k menším koncentracím. Zároveň Rickenmannova transportní rovnice předpovídala menší průtok částic než Marouškova rovnice při stejných podmínkách.

Následně bylo v laboratoři provedeno měření za neustáleného chodu sedimentu. Neustálenost byla vyvolána manipulací klapky ve spodním profilu, což vedlo k erozi dna ve spodní části a zvýšení množství přitékajícího sedimentu na začátek žlabu (protože se jedná o uzavřený okruh). Stejná manipulace byla simulována matematickým modelem změnou okrajové podmínky. Zvolené kombinace transportní a drsnostní rovnice byly dvě (Rickenmannova drsnostní a transportní rovnice, nová drsnostní a Matouškova transportní rovnice). Jak je vidět z podélných profilů zobrazených v časových krocích, u obou modelů souhlasí vypočtená hloubka se změřenou při experimentu. Vypočtená úroveň dna se nepatrně liší od naměřených hodnot. Lepší shody došlo při použití kombinace námi odvozené drsnostní rovnice a Marouškovy transportní rovnice.

Podařilo se sestavit funkční matematický model. Jeho výsledky jsou porovnatelné s měřenými daty a odchylky jsou v přijatelných mezích. Dále bude probíhat práce na odvozování rovnic popisujících přesněji intenzivní chod sedimentu v tocích. Prací na modelu bude zavedení těchto odvozených rovnic při zachování numerické stability.

Klíčová slova

Matematický model, intenzivní chod sedimentu, žlab

Ing. Štěpán Zrostlík[1] stepan.zrostlik@fsv.cvut.cz

[1]Katedra hydrauliky a hydrologie, Stavební fakulta ČVUT, Thákurova 7, 166 29 Praha 6 - Dejvice